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APPENDIX

A GUIDE FOR NEWCOMERS TO AGENT-BASED MODELING IN THE SOCIAL SCIENCES*

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Abstract

This guide provides pointers to introductory readings, software, and other materials to help newcomers become acquainted with agent-based modeling in the social sciences.

Keywords

Agent-based modeling; Complexity; Emergence; Collective behavior; Evolution; Learning; Norms; Markets; Institutional design; Networks.

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1. Purpose of the guide

The purpose of this guide is to suggest a short list of introductory readings to help newcomers become acquainted with *agent-based modeling (ABM)*. Our primary intended audience is graduate students and advanced undergraduate students in the social sciences. Teachers of ABM might also find this guide of use.

Unlike established methodologies such as statistics and mathematics, ABM has not yet developed a widely shared understanding of what a newcomer should learn. For decades, concepts such as the level of significance in statistics and the derivative in mathematics have been common knowledge that newcomers could be expected to learn. We hope that our selected readings will promote a shared understanding of ABM in the social sciences, not only among newcomers to ABM but also among researchers who already use ABM.

As a clarifying note on terminology, although this guide is directed specifically to social scientists, researchers in a wide range of disciplines are now using ABM to study complex systems. When specialized to computational economic modeling, ABM reduces to *Agent-based Computational Economics (ACE)*.

For the convenience of readers, a parallel on-line guide for newcomers to ABM is available at <http://www.econ.iastate.edu/tesfatsi/abmread.htm> that includes links to our suggested readings, as well as demonstration software, as availability permits.

2. Agent-based modeling and the social sciences¹

The social sciences seek to understand not only how individuals behave but also how the interaction of many individuals leads to large-scale outcomes. Understanding a political or economic system requires more than an understanding of the individuals that comprise the system. It also requires understanding how the individuals interact with each other, and how the results can be more than the sum of the parts.

ABM is well suited for this social science objective. It is a method for studying systems exhibiting the following two properties: (1) the system is composed of interacting agents; and (2) the system exhibits *emergent* properties, that is, properties arising from the interactions of the agents that cannot be deduced simply by aggregating the properties of the agents. When the interaction of the agents is contingent on past experience, and especially when the agents continually adapt to that experience, mathematical analysis is typically very limited in its ability to derive the dynamic consequences. In this case, ABM might be the only practical method of analysis.

¹ For more detailed discussions of many of the points raised in this section, see Robert Axelrod, *Complexity of Cooperation* (1997, Princeton University Press, Princeton, NJ), especially pp. 206-221, and Leigh Tesfatsion, "Agent-Based Computational Economics: A Constructive Approach to Economic Theory," in this Handbook.

ABM begins with assumptions about agents and their interactions and then uses computer simulation to generate “histories” that can reveal the dynamic consequences of these assumptions. Thus, ABM researchers can investigate how large-scale effects arise from the micro-processes of interactions among many agents. These agents can represent people (say consumers, sellers, or voters), but they can also represent social groupings such as families, firms, communities, government agencies and nations.

Simulation in general, and ABM in particular, is a third way of doing science in addition to deduction and induction. Scientists use deduction to derive theorems from assumptions, and induction to find patterns in empirical data. Simulation, like deduction, starts with a set of explicit assumptions. But unlike deduction, simulation does not prove theorems with generality. Instead, simulation generates data suitable for analysis by induction. Nevertheless, unlike typical induction, the simulated data come from a rigorously specified set of assumptions regarding an actual or proposed system of interest rather than direct measurements of the real world. Consequently, simulation differs from standard deduction and induction in both its implementation and its goals. Simulation permits increased understanding of systems through controlled computational experiments.

The specific goals pursued by ABM researchers take four forms: empirical, normative, heuristic, and methodological. The goal of *empirical understanding* asks: Why have particular large-scale regularities evolved and persisted, even when there is little top-down control? Examples of such regularities include standing ovations, trade networks, socially accepted monies, mutual cooperation based on reciprocity, and social norms. ABM researchers seek causal explanations grounded in the repeated interactions of agents operating in specified environments. In particular, they ask whether particular types of observed global regularities can be reliably generated from particular types of agent-based models.

A second goal is *normative understanding*: How can agent-based models be used as laboratories for the discovery of good designs? ABM researchers pursuing this objective are interested in evaluating whether designs proposed for social policies, institutions, or processes will result in socially desirable system performance over time. Examples include design of auction systems, voting rules, and law enforcement. The general approach is akin to filling a bucket with water to determine if it leaks. An agent-based world is constructed that captures the salient aspects of a social system operating under the design. The world is then populated with privately motivated agents with learning capabilities and allowed to develop over time. The key issue is the extent to which the resulting world outcomes are efficient, fair, and orderly, despite attempts by these privately motivated agents to gain individual advantage through strategic behavior.

A third goal is *heuristic*: How can greater insight be attained about the fundamental causal mechanisms in social systems? Even if the assumptions used to model a social system are simple, the consequences can be far from obvious if the system is composed of many interacting agents. The large-scale effects of interacting agents are often surprising because it can be hard to anticipate the full consequences of even simple forms

of interaction. For example, one of the earliest and most elegant agent-based models - the city segregation (or “tipping”) model developed by Thomas Schelling (see Section IV.A below) - demonstrates how residential segregation can emerge from individual choices even when everyone is fairly tolerant.

A fourth goal is *methodological advancement*: How best to provide ABM researchers with the methods and tools they need to undertake the rigorous study of social systems through controlled computational experiments? ABM researchers are exploring a variety of ways to address this objective ranging from careful consideration of methodological principles to the practical development of programming and visualization tools.

In summary, ABM applied to social processes uses concepts and tools from social science and computer science. It represents a methodological approach that could ultimately permit two important developments: (1) the rigorous testing, refinement, and extension of existing theories that have proved to be difficult to formulate and evaluate using standard statistical and mathematical tools; and (2) a deeper understanding of fundamental causal mechanisms in multi-agent systems whose study is currently separated by artificial disciplinary boundaries.

3. Selection criteria

We decided at the outset to offer a short list of readings rather than make any attempt at comprehensiveness. We based our selections on two criteria: (i) the educational value of the reading for newcomers to ABM in the social sciences; and (ii) the accessibility of the reading. The specific choice of topics and readings is our own. We recognize that our selections are personal and necessarily somewhat arbitrary.

4. Suggested readings

4.A Complexity and ABM

Vicsek, Tamas (2002), “Complexity: The Bigger Picture,” *Nature*, Vol. 418, p. 131.

In this short essay, Vicsek describes how computer simulation fits into the scientific enterprise. The goal is to “capture the principal laws behind the exciting variety of new phenomena that become apparent when the many units of a complex system interact.”

Callahan, Paul, “What is the Game of Life?”

Accessible online at <http://www.math.com/students/wonders/life/life.html>, this interactive website explains and demonstrates a delightful “game” invented by John Conway in 1970. Although the Game of Life is not an agent-based model, it is a fascinating illustration of how just three simple behavioral rules can lead to extremely complicated outcomes.

Schelling, Thomas C. (1978), *Micromotives and Macrobehavior*, Norton, New York, pp. 137-57.

This classic work demonstrates what can happen when behavior in the aggregate is more than the simple summation of individual behaviors. The highlighted pages present an agent-based model that shows how a high degree of residential segregation can emerge from the location choices of fairly tolerant individuals.

4.B Emergence of collective behavior

Granovetter, Mark (1978), “Threshold Models of Collective Behavior,” *American Journal of Sociology*, Vol. 83, pp. 1420-42.

Threshold models are a class of mathematically tractable models that do not require ABM to determine the global behavior that will emerge from individual choices. In a threshold model, the key specification is each agent's *threshold* for each of its possible actions, i.e., the proportion of other agents who must prefer to take a particular action before the given agent will prefer to take this action. Granovetter develops a threshold model in which each agent has the same two alternative actions and the thresholds for these actions differ across agents. For a given frequency distribution of thresholds, the model calculates the equilibrium number of agents taking each action. One suggested application is to civil violence, in which each agent must decide whether or not to join a riot. It is interesting to compare Granovetter's threshold model outcomes to the richer outcomes obtained for an agent-based model of civil violence in the following article by Joshua Epstein.

Epstein, Joshua M. (2002), “Modeling Civil Violence: An Agent-Based Computational Approach,” *Proceedings of the National Academy of Sciences, U.S.A.*, Vol. 99, pp. 7243-7250.

Epstein uses a spatial agent-based model to explore civil violence. A central authority uses “cops” to arrest (remove) actively rebelling citizens from the society for a specified jail term. In each time step, each agent (cop or citizen) randomly moves to a new unoccupied site within its limited vision. A rebelling citizen's estimated arrest probability is assumed to fall as the ratio of actively rebelling citizens to cops that the citizen perceives in its vicinity increases. Each citizen in each time step decides whether to actively rebel or not depending on this perceived ratio. Epstein shows how the complex dynamics resulting from these simple assumptions can generate empirically interesting macroscopic regularities that are difficult to analyze using more standard modeling approaches.

Cederman, Lars-Erik (2003), “Modeling the Size of Wars,” *American Political Science Review*, Vol. 97, pp. 135-150.

Power-law distributions, scaling laws and self-organized criticality are features of many frequency distributions, from word usage to avalanches, and from firms to cities. A set of events is said to behave in accordance with a *power law distribution* if large events are rarer than small events, and specifically if the frequency of an event is *inversely* proportional to its size. An example is the distribution of the sizes of wars. Cederman uses an agent-based model of war and state formation in the context of

technological change to account for this observed regularity. His paper is a good example of how a fairly complicated model and its implications can be clearly presented, with details left to an appendix.

Miller, John, and Scott E. Page (2004), "The Standing Ovation Problem," *Complexity*, Vol. 9, No. 5, May/June, pp. 8-16.

Miller and Page use audience ovation to introduce many key ABM themes, in particular the emergence of collective behavior, and to provide specific modeling suggestions suitable for implementation by newcomers to the field. As a public performance draws to a close, and audience members begin to applaud and some even tentatively to stand, will a standing ovation ensue or not? This is the famous *Standing Ovation Problem (SOP)* inspired by the seminal work of Thomas Schelling on the relationship between micro decisions and macro behaviors (see Section IV.A above). Miller and Page use the SOP to illustrate how complex social dynamics can arise from the interactions among simple personal choices, in this case to stand or not. They argue (p. 9) that the success of the SOP as an expository device is that it forces modelers "to confront the core methodological issue in complex adaptive social systems, namely, how does one model a system of thoughtful, interacting agents in time and space."

4.C Evolution

Dawkins, Richard (1989), *The Selfish Gene*, New Edition, Oxford University Press, Oxford, UK, pp. 1-45.

If you are going to read only one book on evolution, this delightful and insightful book is a good choice. You will be amazed at the implications of the inclusive fitness perspective.

Sigmund, Karl (1993), *Games of Life: Explorations in Ecology, Evolution, and Behavior*, Oxford University Press, Oxford, UK, pp. 155-206.

Writing in a lively and engaging style, Sigmund provides a non-technical introduction to models of evolution. Topics include population ecology and chaos, random drift and chain reactions, population genetics, evolutionary game theory, and the evolution of cooperation based on reciprocity. The highlighted pages cover the latter two topics, of most relevance to social scientists.

4.D Learning

Clark, Andy (1998), *Being There: Putting Brain, Body, and World Together Again*, The MIT Press, Cambridge, MA, pp. 179-192.

This delightfully written book addresses foundational questions about how people (and robots) can make sense of the confusing world in which they live. The highlighted pages apply this perspective to markets.

Holland, John H. (1992), “Genetic Algorithms,” *Scientific American*, Vol. 267, July, pp. 66-72.

The genetic algorithm is a search technique inspired by the evolutionary effectiveness of mutation and differential reproduction. The algorithm provides a convenient way to model agents of limited rationality that adapt and/or evolve over time. Each agent might be responding to a fixed environment, or to an ever-changing social environment consisting of many agents who are continually adapting to each other. The article by Rick Riolo in the same issue shows how to incorporate a genetic algorithm in one’s own agent-based model.

Vriend, Nicolaas (2000), “An Illustration of the Essential Difference Between Individual and Social Learning, and its Consequence for Computational Analyses,” *Journal of Economic Dynamics and Control*, Vol. 24, pp. 1-19.

Vriend focuses on the importance of the *level* of learning for computational agents. An agent is said to employ *individual-level learning* when it learns from its own past experiences, and to employ *population-level learning* when it learns from other agents, e.g., through mimicry of their observed behaviors. Using a simple market model for concrete illustration, Vriend demonstrates that substantially different outcomes can result when profit-seeking firms use individual-level genetic algorithm learning versus population-level genetic algorithm learning.

4.E Norms

Hofstadter, Douglas (1983), “Computer Tournaments of the Prisoner’s Dilemma Suggest How Cooperation Evolves,” *Scientific American*, May, pp. 18-26.

Hofstadter explains Robert Axelrod’s computer tournaments, which explored the evolution of cooperation in the iterated Prisoner’s Dilemma. For the original work, including agent-based models, formal theorems, and many real-world applications, see Robert Axelrod, *Evolution of Cooperation* (1984, NY: Basic Books).

Axelrod, Robert (1986), “An Evolutionary Approach to Norms,” *American Political Science Review*, Vol. 80, pp. 1095-1111.

This article develops an agent-based model with a simple form of learning using the genetic algorithm to explore what can happen when many agents adapt to each other’s behavior over time. Agents can be more or less bold (say by cheating), and more or less vengeful (say by reporting cheaters). The model shows the conditions under which a collective action problem can be solved by a self-sustaining metanorm: punish those who do not enforce the norm because others might punish you for *not* doing so.

Nowak, Martin A., Karen M. Page, and Karl Sigmund (2000), “Fairness Versus Reason in the Ultimatum Game,” *Science*, Vol. 289, September 8, pp. 1773-75.

The authors consider the *Ultimatum Game* in which two players are offered a chance to win a certain sum of money. One player, the proposer, gets to offer a portion of the sum to the other player, retaining the rest. The second player gets to accept or reject the offer, with rejection resulting in no money for either player. The rational solution,

according to game theory, is for the proposer to offer as little as possible and for the other player to accept. When humans play the game, however, the most frequent offer is an equal (“fair”) share. The authors employ evolutionary dynamics to explain how this “irrational” anchoring on fair shares might have evolved among humans in part through a rational concern for reputation. Specifically, accepting low offers, if generally known and remembered, increases the chances of receiving low offers in subsequent encounters; and making low offers becomes irrational if low offers are not accepted.

Epstein, Joshua M. (2001), “Learning to be Thoughtless: Social Norms and Individual Competition,” *Computational Economics*, Vol. 18, pp. 9-24.

Epstein uses an agent-based model to study experimentally an important observed aspect of social norm evolution: namely, that the amount of time an individual devotes to thinking about a behavior tends to be inversely related to the strength of the social norms that relate to this behavior. In the limit, once a behavioral norm is firmly entrenched in a society, individuals tend to conform to the norm without explicit thought. Epstein's innovative model permits agents to learn how to behave (what behavioral norm to adopt), but it also permits agents to learn how much to think about how to behave.

4.F Markets

Albin, Peter and Duncan K. Foley (1992), “Decentralized, Dispersed Exchange Without an Auctioneer: A simulation study,” *Journal of Economic Behavior and Organization*, Vol. 18, pp. 27-51.

Albin and Foley simulate pure exchange among geographically dispersed utility-seeking traders with endowments of two distinct types of goods, and with bounds to rationality and calculation. Exchange is entirely decentralized. The authors show that this decentralized exchange process achieves a substantial improvement in trader welfare relative to randomly allocated goods.

Gode, D. K., and S. Sunder (1993), “Allocative Efficiency of Markets with Zero Intelligence Traders: Market as a partial substitute for individual rationality,” *Journal of Political Economy*, Vol. 101, pp. 119-137.

Gode and Sunder report on continuous double-auction experiments with computational traders. They find that high market efficiency is generally attained even when the traders randomly select bids and offers from within their budget sets as long as these “zero intelligence” traders abide by certain protocols restricting the order of executed trades. The authors conclude that the high market efficiency typically observed in continuous double-auction experiments with human subjects is due to the structure of the auction and not to learning. Their seminal work has highlighted an important issue now being actively pursued by many other researchers: what are the relative roles of learning and institutional arrangements in the determination of economic, social, and political outcomes?

LeBaron, Blake (2002), “Building the Santa Fe Artificial Stock Market,” Working Paper, Brandeis University, June.

LeBaron provides an insider's look at the construction of the Santa Fe Artificial Stock Market model. He considers the many design questions that went into building the model from the perspective of a decade of experience with agent-based financial markets. He also provides an assessment of the model's overall strengths and weaknesses.

4.G Institutional design

Kollman, Ken, John H. Miller, and Scott E. Page (1997), “Political Institutions and Sorting in a Tiebout Model,” *American Economic Review*, Vol. 87, pp. 977-992.

The authors develop an agent-based model to explore how social outcomes are affected by the political institutions used to aggregate individual choices on local public goods issues, such as whether or not to finance a community swimming pool. Examples of such political institutions are referenda, two-party competition, and proportional representation. For each tested political institution, assumed to be commonly in use across all jurisdictions, citizens “vote with their feet” in each time period regarding which jurisdiction they wish to inhabit. The policy positions resulting in any given jurisdiction depend on the preferences of the citizens located within that jurisdiction, in a manner determined by the political institution in force. Citizens can continue to relocate in response to changing local policy positions, and local policy positions can continue to change in response to citizen relocations. The authors find that social efficiency is highest under political institutions such as two-party competition or proportional representation that initially induce citizens to undertake a suitable degree of experimentation among alternative jurisdictions.

Lansing, Stephen, and James N. Kremer (1993), “Emergent Properties of Balinese Water Temple Networks: Coadaptation on a Rugged Fitness Landscape,” *American Anthropologist*, Vol. 95, pp. 97-114.

Over hundreds of years, Balinese farmers have developed an intricate hierarchical network of “water temples” dedicated to agricultural deities in parallel with physical transformations of their island deliberately undertaken to make it more suitable for growing irrigated rice. The water temple network plays an instrumental role in the coordination of activities related to rice production. Representatives of different water temple congregations meet regularly to decide cropping patterns, planting times, and water usage, thus helping to synchronize harvests and control pest populations. Lansing and Kremer develop an ecological simulation model to illuminate the system-level effects of the water temple network, both social and ecological. Their anthropological study illustrates many important ABM concepts, including emergent properties, fitness landscapes, co-adaptation, and the effects of different institutional designs.

Simon, Herbert (1982), “The Architecture of Complexity,” pp. 193-230 in *The Sciences of the Artificial*, Second Edition, The MIT Press, Cambridge, MA.

Simon informally defines a “complex system” to be a system made up of a large number of parts that interact in a non-simple way. He considers a number of complex systems encountered in the behavioral sciences, from families to formal organizations, and describes features that are common in a wide variety of such systems. His central theme (p. 196) is that “complexity frequently takes the form of hierarchy and that

hierarchic systems have some common properties independent of their specific content.” He discusses the design advantages of nearly decomposable subsystems with a hierarchical organization of their parts. He also conjectures that complex systems evolve from simple systems much more rapidly if there are stable intermediate forms along the way, hence evolution favors hierarchic over non-hierarchic systems.

4.H Networks

Wilhite, Allen (2001), “Bilateral Trade and ‘Small-World’ Networks,” *Computational Economics*, Vol. 18, No. 1, pp. 49-64.

Wilhite develops an agent-based computational model of a bilateral exchange economy. He uses this model to explore the consequences of restricting trade to different types of networks, including a “small-world network” with both local connectivity and global reach. His key finding is that small-world networks provide most of the market-efficiency advantages of completely connected networks while retaining almost all of the transaction cost economies of locally connected networks.

Kirman, Alan P., and Nicolaas J. Vriend (2001), “Evolving Market Structure: An ACE Model of Price Dispersion and Loyalty,” *Journal of Economic Dynamics and Control*, Vol. 25, Nos. 3-4, pp. 459-502.

Social scientists typically study the implications of *given* interaction networks, e.g., friendship or trade networks. An important aspect of many social systems, however, is how agents come to *form* interaction networks. Kirman and Vriend address this issue in the context of an agent-based computational model capturing salient structural aspects of the actual wholesale fish market in Marseilles, France. Two features characterizing this actual market are: (a) loyalty relationships (persistent trade partnerships) between particular buyers and sellers; and (b) persistent price dispersion unexplainable by observable characteristics of the fish. The simulation results show that loyalty relationships can indeed emerge naturally between particular buyer-seller pairs as the buyers and sellers co-evolve their trading rules over time. Buyers learn to become loyal to particular sellers while, at the same time, sellers learn to offer higher payoffs (lower prices and more reliable supplies) to their more loyal buyers. Moreover, this evolving trade network supports persistent price dispersion over time.

4.I Modeling techniques

Macy, Michael W., and Robert Willer (2002), “From Factors to Actors: Computational Sociology and Agent-Based Modeling,” *Annual Review of Sociology*, Vol. 28, pp. 143-166.

While written for sociologists, this review article should be of value to all agent-based modelers. It places ABM in its historical context, explains its meaning and goals, provides many good examples, and offers useful advice to those who want to try it for themselves. Other articles with explicit modeling advice include LeBaron (2002) and Miller and Page (2004) cited above.

5. What to do next

Browse the comprehensive **website** at <http://www.econ.iastate.edu/tesfatsi/ace.htm> to find agent-based researchers in your neck of the woods, links to specific topics in ABM, course syllabi, demonstration software, and much more.

Use the chapters in this **handbook** to help you explore specific topics.

Explore the **journals** that publish a good deal of ABM: *Journal of Artificial Societies and Social Simulation* (on-line); *Computational Economics*; *Journal of Economic Behavior and Organization*; *Games and Economic Behavior*; *Journal of Economic Dynamics and Control*; and *Complexity*. For weekly news items, including upcoming conferences, see the *Complexity Digest* (on line).²

Master the **mathematical and statistical tools** that are commonly used in ABM by studying basic mathematical analysis (especially probability theory and non-linear dynamics), game theory, and elementary statistics (e.g., hypothesis testing and regression).

Learn a **programming language** so that you can try your hand at building and running your own agent-based models. For younger beginners, we recommend StarLogo.³ For older beginners, we recommend a language with object-oriented capabilities such as Java, C++, or C#,⁴ supplemented with an agent-based toolkit (see below). Another possibility is Matlab, which is steadily increasing its ABM capabilities.

Explore the **agent-based toolkits** that are available to assist agent-based modelers with common tasks such as constructing agents and displaying output in the form of tables, charts, graphs, and movies.⁵ For example, *Repast* is specifically designed for agent-based modeling in the social sciences and supports model development in many different programming languages and on virtually all modern computing platforms.⁶ Another possibility is *NetLogo*, a cross-platform multi-agent

² See <http://www.econ.iastate.edu/tesfatsi/publish.htm> for links to these journals as well as to many other journals and book publishers that support the publication of ABM-related work.

³ StarLogo is a programmable modeling environment for exploring the workings of decentralized systems that has been specifically designed to be user-friendly for K-12 students. Extensive support materials for StarLogo can be found at <http://education.mit.edu/starlogo/>.

⁴ ABM is increasingly being implemented using languages with object-oriented programming (OOP) capabilities, such as Java, C++, and C#. A good introduction to OOP is Matt Weisfeld, *The Object-Oriented Thought Process* (2000, SAMS Publishing, Division of Macmillan, Indianapolis, Indiana). This book is designed to help newcomers learn OOP guidelines for solid class design and master the major OOP concepts of inheritance, composition, interfaces, and abstract classes. The author motivates and illustrates his points by taking readers step by step through simple concrete examples.

⁵ See <http://www.econ.iastate.edu/tesfatsi/acecode.htm> for annotated pointers to a wide variety of programming languages and agent-based toolkits currently being used for ABM.

⁶ See <http://www.econ.iastate.edu/tesfatsi/repastsg.htm> for detailed information about Repast.

programming environment.⁷ Both Repast and NetLogo are actively maintained and freely available, and their relative ease of use has attracted growing communities of users.

Explore **special journal issues** devoted to agent-based modeling and related themes.

These include: *American Behavioral Scientist* (Vol. 42, August 1999); *Science* (Vol. 284, April 1999); *Journal of Economic Dynamics and Control* (Vol. 25, Nos. 3-4, 2001), *Computational Economics* (Vol. 18, No. 1, 2001); and the *Proceedings of the National Academy of Sciences, USA* (Vol. 99, Supplement 3, 2002).⁸

For a wonderful **introduction to computational aspects of complex systems**, including fractals, chaos, cellular automata, neural networks, and a helpful glossary of terms, we highly recommend Gary William Flake, *The Computational Beauty of Nature* (1998, MIT Press, Cambridge, MA).⁹

⁷ See <http://ccl.northwestern.edu/netlogo> for detailed information about NetLogo.

⁸ See <http://www.econ.iastate.edu/tesfatsi/avolumes.htm> for an annotated list of special ABM-related journal issues together with volumes of ABM-related readings.

⁹ See <http://mitpress.mit.edu/books/FLAOH/cbnhtml/> for detailed information and supporting materials for Flake's book.